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INTEGRAL SOLUTIONS OF BINARY QUADRATIC DIOPHANTINE EQUATION $x^2-3xy+y^2+12x=0$

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ABSTRACT
The binary quadratic Diophantine equation given by $x^2-3xy+y^2+12x=0$ is analyzed for its patterns of non-zero distinct integral solutions. A few interesting relations between the solutions and special polygonal numbers are exhibited.

KEYWORDS: Binary Quadratic, integral solutions, polygonal numbers.

I. INTRODUCTION
Binary quadratic Diophantine equations are rich in variety [1-3].For an extensive review of sizable literature and various problems, one may refer [1-16]. In this communication, we consider yet another interesting ternary quadratic equation $x^2-3xy+y^2+12x=0$ and obtain infinitely many non-trivial integral solutions. A few interesting relations between the solutions and special Polygonal numbers are presented.

Notations Used
- $t_{m,n}$- Polygonal number of rank 'n' with size 'm'
- P_n^m- Pyramidal number of rank 'n' with size 'm'

II. METHOD OF ANALYSIS
The Diophantine equation representing the binary quadratic equation to be solved for its non zero distinct integral solution is

$$x^2-3xy+y^2+12x=0 \tag{1}$$

Note that (1) is satisfied by the following non-zero integer pairs

(-12,-36),(12,12),(12,24)

However, we have other solutions for (1), which are illustrated below

Solving (1) for x, we have

$$x^2+(-3y+12)x+y^2=0$$

$$x=\frac{1}{2}\left(3y-12\pm\sqrt{(-3y+12)^2-4y^2}\right)$$

$$x=\frac{1}{2}\left(3y-12\pm\sqrt{(5y^2-72y+12^2)}\right) \tag{2}$$

Let

$$\alpha^2=5y^2-72y+12^2$$

Which is written as

$$(5y-36)^2=5\alpha^2+24^2$$

$$Y^2=5\alpha^2+24^2 \tag{3}$$

Where $Y=5y-36$ (4)

The least positive integer solution of (3) is

$$\alpha_0=12,\qquad Y_0=36$$

Now to find the other solution of (3), consider the Pellian equation

$$Y^2=5\alpha^2+1 \tag{5}$$

Whose fundamental solution is $(\widetilde{\alpha_0},\widetilde{Y_0})$ = (4,9)

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The general solution of (5) is of the form

$$\widetilde{Y_s} + \sqrt{D}\widetilde{\alpha_s} = \left(\widetilde{Y_0} + \sqrt{D}\,\widetilde{\alpha_0}\right)^{s+1}$$

$$\widetilde{Y_s} + \sqrt{5}\widetilde{\alpha_s} = \left(9 + 4\sqrt{5}\,\right)^{s+1} \qquad (6)$$

And

$$\widetilde{Y_s} - \sqrt{5}\widetilde{\alpha_s} = \left(9 - 4\sqrt{5}\,\right)^{s+1} \qquad (7)$$

$$(6) + (7) \Longrightarrow \quad \widetilde{Y_s} + \sqrt{5}\,\widetilde{\alpha_s} + \widetilde{Y_s} - \sqrt{5}\,\widetilde{\alpha_s} = \left(9 + 4\sqrt{5}\,\right)^{s+1} + \left(9 - 4\sqrt{5}\,\right)^{s+1}$$

$$\widetilde{Y_s} = \frac{1}{2}\left[\left(9 + 4\sqrt{5}\,\right)^{s+1} + \left(9 - 4\sqrt{5}\,\right)^{s+1}\right]$$

$$(6) - (7) \Longrightarrow \quad \widetilde{Y_s} + \sqrt{5}\,\widetilde{\alpha_s} - \widetilde{Y_s} + \sqrt{5}\,\widetilde{\alpha_s} = \left(9 + 4\sqrt{5}\,\right)^{s+1} - \left(9 - 4\sqrt{5}\,\right)^{s+1}$$

$$\widetilde{\alpha_s} = \frac{1}{2\sqrt{5}}\left[\left(9 + 4\sqrt{5}\,\right)^{s+1} - \left(9 - 4\sqrt{5}\,\right)^{s+1}\right]$$

Then

$$\left.\begin{aligned} \widetilde{Y_s} &= \frac{f_s}{2} \\ \widetilde{\alpha_s} &= \frac{g_s}{2\sqrt{5}} \end{aligned}\right\} \qquad (8)$$

Where

$$f_s = \left[\left(9 + 4\sqrt{5}\,\right)^{s+1} + \left(9 - 4\sqrt{5}\,\right)^{s+1}\right]$$

$$g_s = \left[\left(9 + 4\sqrt{5}\,\right)^{s+1} - \left(9 - 4\sqrt{5}\,\right)^{s+1}\right]$$

$$s = 0, 2, 4 \ldots \ldots$$

Applying the lemma of Brahmaguptha between $\left(\widetilde{\alpha_0},\, \widetilde{Y_0}\right)$ & $(\widetilde{\alpha_s},\, \widetilde{Y_s})$

The other solutions of (3) can be obtained from the relations

$$\begin{aligned} \alpha_{s+1} &= \alpha_0\widetilde{Y_s} + Y_0\widetilde{\alpha_s} \\ &= 12\widetilde{Y_s} + 36\,\widetilde{\alpha_s} \\ &= 12\frac{f_s}{2} + 36\,\frac{g_s}{2\sqrt{5}} \end{aligned}$$

$$\alpha_{s+1} = 6f_s + \frac{18}{\sqrt{5}}\,g_s \qquad (9)$$

$$\begin{aligned} Y_{s+1} &= Y_0\widetilde{Y_s} + D\alpha_0\widetilde{\alpha_s} \\ &= 36\widetilde{Y_s} + 5 * 12\,\widetilde{\alpha_s} \\ &= 36\frac{f_s}{2} + 5 * 12\,\frac{g_s}{2\sqrt{5}} \end{aligned}$$

$$Y_{s+1} = 18f_s + 6\sqrt{5}\,g_s \qquad (10)$$

Using $(4), (6), (7), (9)$ *and* (10), the non zero distinct integer solution of the hyperbola (1) are obtained as follows

$$x_{s+1} = \frac{1}{2}(3Y_{s+1} - 12 \pm \alpha_{s+1}) \qquad (11)$$

$$y_{s+1} = \frac{1}{5}(Y_{s+1} + 36)$$

The recurrence relations satisfied by x_{s+1}, y_{s+1} are respectively

$$x_{s+1} - 322x_{s+3} + x_{s+5} + 1536 = 0$$

$$y_{s+1} - 322y_{s+3} + y_{s+5} + 2304 = 0$$

For simplicity considering positive sign on the R.H.S of (11) a few numerical examples are given in table

S	x_{s+1}	y_{s+1}
0	300	120
1	95052	36312
2	30604908	116900490

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Observations

1. $x_0 - y_0$ is a nasty number
2. $x_{s+1} - y_{s+1} \equiv x_{s+1} + y_{s+1} (mod\ 8)$
3. $x_{s+1} = y_{s+1} (mod\ 9)$
4. Each of the expressions $x_s x_{s+1}$ and $x_{s+1} y_{s+1} + 100$ is a perfect square
5. $x_{s+1} - y_{s+1} \equiv x_{s+1} (mod\ 8)$

III CONCLUSION

In this paper, we have presented four different patterns of non- zero distinct integer solutions of quadratic Diophantine equation $x^2 - 3xy + y^2 + 12x = 0$ and relations between solutions and special numbers are also obtained. To conclude, one may search for other patterns of solutions and their corresponding properties.

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